

Chapter 31: Nuclear Applications

Practice Problems

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1. The carbon isotope, $^{12}_6\text{C}$, has a nuclear mass of 12.0000 u.

a. Calculate its mass defect.

$$\begin{array}{rcl} 6 \text{ protons} & = & (6)(1.007825 \text{ u}) = 6.046950 \text{ u} \\ 6 \text{ neutrons} & = & (6)(1.008665 \text{ u}) = 6.051990 \text{ u} \\ & & \text{total } 12.098940 \text{ u} \\ \text{mass of carbon nucleus} & - & 12.000000 \text{ u} \\ \text{mass defect} & = & -0.098940 \text{ u} \end{array}$$

b. Calculate its binding energy in MeV.

$$\begin{aligned} & -(0.098940 \text{ u})(931.49 \text{ MeV/u}) \\ & = -92.162 \text{ MeV} \end{aligned}$$

2. The isotope of hydrogen that contains 1 proton and 1 neutron is called deuterium. The mass of its nucleus is 2.0140 u.

a. What is its mass defect?

$$\begin{array}{rcl} 1 \text{ proton} & = & 1.007825 \text{ u} \\ 1 \text{ neutron} & = & 1.008665 \text{ u} \\ & & 2.016490 \text{ u} \\ \text{mass of deuterium nucleus} & = & -2.0140 \text{ u} \\ \text{mass defect} & = & -0.0025 \text{ u} \end{array}$$

b. What is the binding energy of deuterium in MeV?

$$-(0.0025 \text{ u})(931.49 \text{ MeV/u}) = -2.3 \text{ MeV}$$

3. A nitrogen isotope, $^{15}_7\text{N}$, has 7 protons and 8 neutrons. Its nucleus has a mass of 15.00011 u.

a. Calculate the mass defect of this nucleus.

$$\begin{array}{rcl} 7 \text{ protons} & = & 7(1.007825 \text{ u}) = 7.054775 \text{ u} \\ 8 \text{ neutrons} & = & 8(1.008665 \text{ u}) = 8.069320 \text{ u} \\ & & \text{total } 15.124095 \text{ u} \\ \text{mass of nitrogen nucleus} & - & 15.00011 \text{ u} \\ \text{mass defect} & = & -0.123985 \text{ u} \\ & = & -0.12399 \text{ u} \end{array}$$

Practice Problems

b. Calculate the binding energy of the nucleus.

$$-(0.12399 \text{ u})(931.49 \text{ MeV/u}) = -115.50 \text{ MeV}$$

4. An oxygen isotope, $^{16}_8\text{O}$, has a nuclear mass of 15.99491 u.

a. What is the mass defect of this isotope?

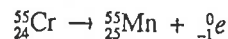
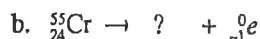
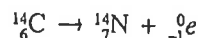
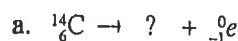
$$\begin{array}{rcl} 8 \text{ protons} & = & (8)(1.007825 \text{ u}) = 8.062600 \text{ u} \\ 8 \text{ neutrons} & = & (8)(1.008665 \text{ u}) = 8.069320 \text{ u} \\ & & \text{total } 16.131920 \text{ u} \\ \text{mass of oxygen nucleus} & - & 15.99491 \text{ u} \\ \text{mass defect} & = & -0.13701 \text{ u} \end{array}$$

b. What is the binding energy of its nucleus?

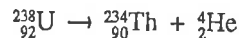
$$-(0.13701 \text{ u})(931.49 \text{ MeV/u}) = -127.62 \text{ MeV}$$

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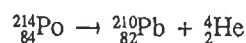
5. Use Table C-5 of the Appendix to complete the following nuclear equations.



6. Write the nuclear equation for the transmutation of a uranium isotope, $^{238}_{92}\text{U}$, into a thorium isotope, $^{234}_{90}\text{Th}$, by emission of an alpha particle.

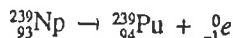
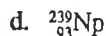
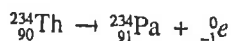
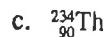
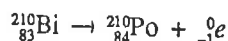
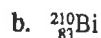
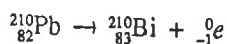
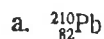


7. A radioactive polonium isotope, $^{214}_{84}\text{Po}$, undergoes alpha decay and becomes lead. Write the nuclear equation.



Practice Problems

8. Write the nuclear equations for the beta decay of these isotopes.



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9. a. Calculate the mass defect for the deuterium-tritium fusion reaction used in the Tokamak, $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0\text{n}$.
- Input masses $2.014102 \text{ u} + 3.016049 \text{ u}$
 $= 5.030151 \text{ u}$
 Output masses $4.002603 \text{ u} + 1.008665 \text{ u}$
 $= 5.011268 \text{ u}$
 Difference is -0.018883 u
- b. Find the energy equivalent of the mass defect.
- $-(0.018883 \text{ u})(931.49 \text{ MeV/u})$
 $= -17.589 \text{ MeV}$
10. Calculate the energy released for the overall reaction in the sun where four protons produce one ^4_2He , two positrons, and two neutrinos.

Practice Problems

Positron mass

$$= (9.109 \times 10^{-31} \text{ kg}) \left[\frac{1 \text{ u}}{1.661 \times 10^{-27} \text{ kg}} \right]$$

$$= 0.000548 \text{ u}$$

$$\text{Input mass: } 4 \text{ protons} = 4(1.007825 \text{ u})$$

$$= 4.031300 \text{ u}$$

Output mass:

$$^4_2\text{He} + 2 \text{ positrons} = 4.002603 \text{ u} + 2(0.000548 \text{ u})$$

$$= 4.003699$$

$$\text{Mass difference} = 0.027601 \text{ u}$$

$$\text{Energy released} = (0.027601 \text{ u})(931.49 \text{ MeV/u})$$

$$= 25.710 \text{ MeV}$$

Chapter Review Problems

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1. A carbon isotope, $^{13}_6\text{C}$, has a nuclear mass of 13.00335 u .
- a. What is the mass defect of this isotope?
- 6 protons $= (6)(1.007825 \text{ u}) = 6.046950 \text{ u}$
 7 neutrons $= (7)(1.008665 \text{ u}) = 7.060655 \text{ u}$
 total 13.107605 u
 mass of carbon nucleus $- 13.00335 \text{ u}$
 mass defect -0.10426 u
- b. What is the binding energy of its nucleus?
- $-(0.10426 \text{ u})(931 \text{ MeV/u}) = -97.1 \text{ MeV}$
2. A nitrogen isotope, $^{14}_7\text{N}$, has a nuclear mass of approximately 14.00307 u .
- a. What is the mass defect of the nucleus?
- 7 protons $= (7)(1.007825 \text{ u}) = 7.054775 \text{ u}$
 7 neutrons $= (7)(1.008665 \text{ u}) = 7.060655 \text{ u}$
 total 14.11543 u
 mass of nitrogen nucleus $- 14.00307 \text{ u}$
 mass defect -0.11236 u

Chapter Review Problems

- b. What is the binding energy of this nucleus?

$$-(0.11236 \text{ u})(931 \text{ MeV/u}) = -105 \text{ MeV}$$

- c. What is the binding energy per nucleon?

$$\begin{aligned} \text{Energy} &= \frac{-105 \text{ MeV}}{14 \text{ nucleons}} \\ &= -7.5 \text{ MeV/nucleon} \end{aligned}$$

3. A nitrogen isotope, $^{12}_7\text{N}$, has a nuclear mass of 12.0188 u.

- a. What is the binding energy per nucleon?

$$\begin{aligned} 7 \text{ protons} &= (7)(1.007825 \text{ u}) = 7.054775 \text{ u} \\ 5 \text{ neutrons} &= (5)(1.008665 \text{ u}) = 5.043325 \text{ u} \\ \text{total} &= 12.098100 \text{ u} \\ \text{mass of nitrogen-12 nucleus} &= 12.0188 \text{ u} \\ \text{mass defect} &= -0.0793 \text{ u} \end{aligned}$$

binding energy

$$-(0.0793 \text{ u})(931 \text{ MeV/u}) = -73.8 \text{ MeV}$$

$$\begin{aligned} \text{binding energy per nucleon} &= \frac{-73.8 \text{ MeV}}{12 \text{ nucleons}} \\ &= -6.15 \text{ MeV} \end{aligned}$$

- b. Does it require more energy to separate a nucleon from a $^{14}_7\text{N}$ nucleus or from a $^{12}_7\text{N}$ nucleus? Refer to the previous problem.

It requires more energy to remove a nucleon from nitrogen - 14, 7.5 MeV.

4. The two positively-charged protons in a helium nucleus are separated by about $2.0 \times 10^{-15} \text{ m}$. Use Coulomb's law to find the electric force of repulsion between the two protons. The result will give you an indication of the strength of the strong nuclear force.

$$\begin{aligned} F &= K \frac{qq'}{d^2} \\ &= \frac{K(1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{(2.0 \times 10^{-15} \text{ m})^2} \\ &= 58 \text{ N} \end{aligned}$$

Chapter Review Problems

5. A $^{232}_{90}\text{U}$ nucleus, mass = 232.0372 u, decays to $^{228}_{90}\text{Th}$, mass = 228.0287 u, by emitting an α particle, mass = 4.0026 u, with a kinetic energy of 5.3 MeV. What must be the kinetic energy of the recoiling thorium nucleus?

mass defect

$$\begin{aligned} &= (232.0372 \text{ u}) - (228.0287 \text{ u} + 4.0026 \text{ u}) \\ &= 0.0059 \text{ u} \end{aligned}$$

$$\begin{aligned} \text{total KE} &= (0.0059 \text{ u})(931 \text{ MeV/u}) \\ &= 5.5 \text{ MeV} \end{aligned}$$

KE for thorium nucleus

$$\begin{aligned} &= (5.5 \text{ MeV}) - (5.3 \text{ MeV}) \\ &= 0.2 \text{ MeV} \end{aligned}$$

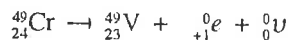
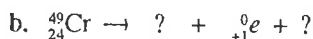
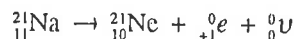
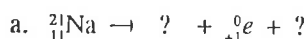
6. The binding energy for ^4_2He is 28.3 MeV.

Calculate the mass of a helium nucleus in atomic mass units.

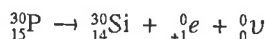
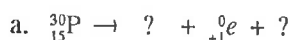
$$\text{Mass defect} = \frac{28.3 \text{ MeV}}{931 \text{ MeV/u}} = 0.0304 \text{ u}$$

$$\begin{aligned} 2 \text{ protons} &= (2)(1.007825 \text{ u}) = 2.015650 \text{ u} \\ 2 \text{ neutrons} &= (2)(1.008665 \text{ u}) = 2.017330 \text{ u} \\ \text{mass} &= 4.032980 \text{ u} \\ \text{minus mass defect} &= -0.0304 \text{ u} \\ &= 4.0026 \text{ u} \end{aligned}$$

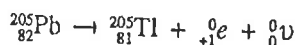
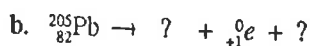
7. The radioactive nucleus indicated in each equation disintegrates by emitting a positron. Complete each nuclear equation.



8. Complete the nuclear equations for these transmutations.



Chapter Review Problems

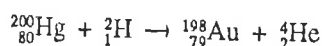


9. A mercury isotope, $^{200}_{80}\text{Hg}$, is bombarded the deuterons, ${}^2_1\text{H}$. The mercury nucleus absorbs the duetron and then emits an alpha particle.

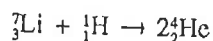
- a. What element is formed by this reaction?

Gold.

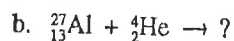
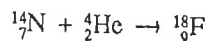
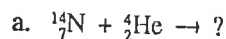
- b. Write the nuclear equation for the reaction.



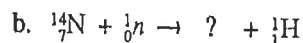
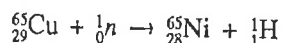
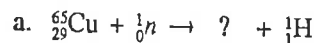
10. When bombarded by protons, a lithium isotope, ${}^7_3\text{Li}$, absorbs a proton and then ejects two alpha particles. Write the nuclear equation for this reaction.



11. Each of the nuclei given below can absorb an alpha particle. Assume that no secondary particles are emitted by the nucleus. Complete each equation.



12. In each of these reactions, a neutron is absorbed by a nucleus. The nucleus then emits a proton. Complete the equations.



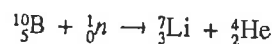
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13. When a boron isotope, $^{10}_5\text{B}$, is bombarded with neutrons, it absorbs a neutron and then emits an alpha particle.

- a. What element is also formed?

Lithium.

- b. Write the nuclear equation for this reaction.

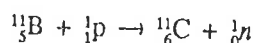


14. When a boron isotope, $^{11}_5\text{B}$, is bombarded with protons, it absorbs a proton and emits a neutron.

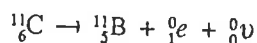
- a. What element is formed?

Carbon.

- b. Write the nuclear equation for this reaction.



- c. The isotope formed is radioactive, decaying by emitting a positron. Write the complete nuclear equation for this reaction.

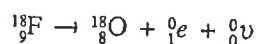


15. The isotope most commonly used in PET scanners is $^{18}_9\text{F}$.

- a. What element is formed by the positron emission of this element?

Oxygen.

- b. Write the equation for this reaction.



- c. The half-life of $^{18}_9\text{F}$ is 110 min. A solution containing 10.0 mg of this isotope is injected into a patient at 8:00 a.m. How much remains at 3:30 p.m.?

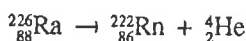
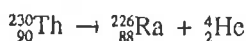
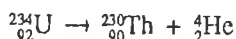
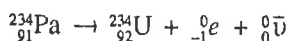
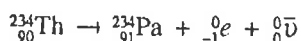
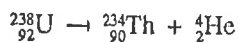
The time is about 4 half-lives, so

$$\frac{1}{16}(10.0 \text{ mg}) \text{ or } 0.63 \text{ mg remains.}$$

Chapter Review Problems

16. The basements of many homes contain the rock granite or are built over granite. Granite contains small amounts of radioactive uranium and thorium. Uranium-238 goes through a series of decays before reaching a stable lead isotope. $^{238}_{92}\text{U}$ emits an alpha, leaving an isotope

that decays by emitting a beta. The next decay is again a β , followed by three α decays. The result is the gas radon that can seep into the basement. Write the equations for these decays.



17. The first atomic bomb released an energy equivalent of 20 kilotons of TNT. One kiloton of TNT is the equivalent of 5.0×10^{12} J. What was the mass of the uranium-235 that was fissioned to produce this energy?

$$\text{Energy} = (20 \text{ kilotons})(5.0 \times 10^{12} \text{ J/kiloton}) \\ = 1.0 \times 10^{14} \text{ J}$$

$$\frac{1.0 \times 10^{14} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 6.3 \times 10^{32} \text{ eV}$$

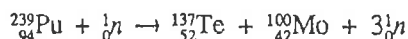
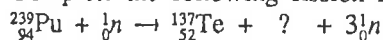
$$\frac{(6.3 \times 10^{32} \text{ eV})}{(2.0 \times 10^2 \text{ MeV/atom})(10^6 \text{ eV/MeV})}$$

$$= 3.1 \times 10^{24} \text{ atoms}$$

$$\frac{3.1 \times 10^{24} \text{ atoms}}{6.02 \times 10^{23} \text{ atoms/mole}} = 5.2 \text{ moles}$$

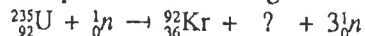
$$(5.2 \text{ moles})(0.235 \text{ kg/mole}) = 1.2 \text{ kg}$$

18. Complete the following fission reaction.

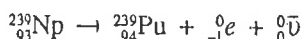
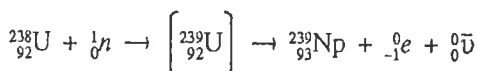
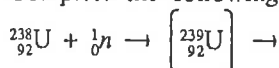


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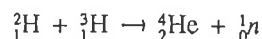
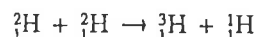
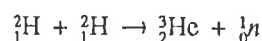
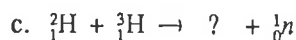
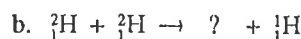
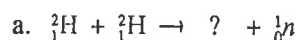
19. Complete the following fission reaction.



20. Plutonium is formed in two steps. First U-238 absorbs a neutron which then becomes unstable U-239. The U-239 emits a beta particle forming a new element and this new element emits a second beta particle forming plutonium. Complete the following reaction.



21. Complete each of the following fusion reactions.



22. One fusion reaction is $^2_1\text{H} + ^2_1\text{H} \rightarrow ^4_2\text{He}$.

- a. What energy is released in this reaction?

$$(2)(2.01410 \text{ u}) - (4.00263 \text{ u}) = 0.02557 \text{ u} \\ (0.02557 \text{ u})(931 \text{ MeV/u}) = 23.8 \text{ MeV}$$

- b. Deuterium exists as a diatomic, two-atom, molecule. One mole of deuterium contains 6.02×10^{23} molecules. Find the amount of energy released, in joules, in the fusion of one mole of deuterium molecules.

$$(23.8 \text{ MeV/molecule}) \\ \times (6.02 \times 10^{23} \text{ molecules/mole}) \\ \times (1.6 \times 10^{-19} \text{ J/eV})(10^6 \text{ eV/MeV}) \\ = 2.29 \times 10^{12} \text{ J/mole}$$

Chapter Review Problems

- c. When one mole of deuterium burns, it releases 2.9×10^6 J. How many moles of deuterium molecules would have to burn to release just the energy released by the fusion of one mole of deuterium molecules?

$$\frac{2.29 \times 10^{12} \text{ J}}{2.9 \times 10^6 \text{ J/mole}} = 7.9 \times 10^5 \text{ moles}$$

23. The energy released in the fission of one atom of $^{235}_{92}\text{U}$ is 2.00×10^2 MeV. One mole of uranium atoms, 6.02×10^{23} atoms, has a mass of 0.235 kg.

- a. How many atoms are in 1.00 kg $^{235}_{92}\text{U}$?

$$\frac{(6.02 \times 10^{23} \text{ atoms/mole})(1.00 \text{ kg})}{(0.235 \text{ kg/mole})} = 2.56 \times 10^{24} \text{ atoms.}$$

- b. How much energy would be released if all the atoms in 1.00 kg of $^{235}_{92}\text{U}$ underwent fission?

$$(2.56 \times 10^{24} \text{ atoms/kg})(2.0 \times 10^2 \text{ MeV/atom})(1.6 \times 10^{-19} \text{ J/eV})(10^6 \text{ eV/MeV}) = 8.2 \times 10^{13} \text{ J}$$

- c. A typical large nuclear reactor produces fission energy at a rate of 3600 MW. How many kilograms of $^{235}_{92}\text{U}$ are used each second?

$$\frac{3.6 \times 10^9 \text{ J/s}}{8.2 \times 10^{13} \text{ J/kg}} = 4.4 \times 10^{-5} \text{ kg/s}$$

- d. How much $^{235}_{92}\text{U}$ would be used in one year?

$$(4.4 \times 10^{-5} \text{ kg/s})(3.15 \times 10^7 \text{ s/y}) = 1.4 \times 10^3 \text{ kg/y}$$

Supplementary Problems (Appendix B)

1. The carbon isotope, $^{12}_6\text{C}$, has a nuclear mass of 12.000 000 u.

- a. What is the mass defect of this isotope?

$$^{12}_6\text{C} \quad 6(^1_1\text{p}) = 6(1.007\,825 \text{ u}) = 6.046\,950 \text{ u}$$

$$6(^1_0\text{n}) = 6(1.008\,665 \text{ u}) = 6.051\,990 \text{ u}$$

$$\text{total mass of nucleus} \quad 12.098\,940 \text{ u}$$

$$^{12}_6\text{C} \quad - 12.000\,000 \text{ u}$$

$$\text{mass defect} \quad - 0.098\,940 \text{ u}$$

- b. What is the binding energy of its nucleus?

$$E = (931.5 \text{ MeV/u})(0.098\,940 \text{ u})$$

$$= 92.16 \text{ MeV}$$